Relative velocity fluctuations in turbulence

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(Received 2 May 1994)

Photon correlation spectroscopy has been used to investigate grid generated turbulence. A measurement of the probability distribution function for the relative velocity fluctuations provides evidence to our theoretical model that this probability distribution follows a product of a Gaussian- and a Lorentzian-type function.

PACS number(s): 47.27.−i

Light scattered from a seeded fluid flowing through a grid has been investigated both theoretically and experimentally at intermediate Reynolds numbers [1–3]. This scattered light is detected by a photomultiplier tube (PMT) and then sent to a digital correlator to measure the autocorrelation function (ACF) [4,5]. This PMT records the beating of Doppler shifted light from pairs of correlated particles and is modulated at a frequency difference proportional to the velocity difference between the pairs of particles separated by a distance R.

In a turbulent flow, eddies are formed which exist over certain length scales, from the largest allowed by experimental constraints (e.g., mesh size of the grid generating the turbulence) to the smallest eddy determined by viscous dissipation. Further, in these eddies the individual particles can have a range of velocities which are neither constant nor independent of each other, but are correlated due to the nature of the turbulent flow, the correlation depending upon the distance R between the particles.

If we take v_1 and v_2 as the velocities of a pair of particles separated by a distance R in the fluid, then $V(R)=v_1-v_2$ will be the velocity difference between these two particles at the same instant of time. Since large number of particles are under observation at the same time in the observation volume these individual velocities v_1 and v_2 can be taken to be Gaussian by virtue of the central limit theorem [6]. Now in a fluid flow v_1 and v_2 are not generally independent of each other but have some correlation. In a laminar flow $v_1-v_2=V$, and is independent of R. But in a turbulent flow, if we take v_1 and v_2 to be correlated Gaussian, then, as is well known in the theorem relating to Gaussian random variables that the sum or difference of Gaussian random variables is also a Gaussian random variable. So if

$$p(v_1)=\frac{1}{\sqrt{2\pi}\sigma_{v_1}}\exp\left[-\frac{(v_1-\bar{v}_1)^2}{2\sigma_{v_1}^2}\right] \tag{1}$$

and similarly for $p(v_2)$. $\sigma_{v_1},\sigma_{v_2}$ are the variances and $\bar{v}_1$ and $\bar{v}_2$ are the means [6]. Thus

$$P(V(R))=\frac{\exp\left[-\dfrac{V^2}{4(1+\rho)\sigma^2}\right]}{\sqrt{2\pi}\sqrt{2(1+\rho)\sigma^2}}\ , \tag{2}$$

where it has been assumed that

$$\bar{v}_1=\bar{v}_2=0\ ,\quad \sigma_{v_1}^2=\sigma_{v_2}^2=\sigma^2\ .$$

ρ is the correlation coefficient of joint Gaussian random variables v_1 and v_2. In general the correlation coefficient ρ is not fixed and may take a range of values. To take this variation into account, we define a distribution of the ρ which, when incorporated into $P(V(R))$ with the proper weight factor, will give us the correct distribution function. Now it is known [6] that for isotropic scattering process (e.g., rotating ground glass, turbulent flow, etc.), the probability density of the cross section when recorded on a plane is a Cauchy or Lorentzian distribution function. This suggests a Lorentzian distribution for ρ itself of the type

$$p(\rho)=\frac{N}{a^2+\rho^2}\ ,$$

where N is the normalization constant and a is the half-width.

Therefore the final probability distribution of the velocity difference of two Lorentzian correlated velocities is $P_\rho(V(R))$,

$$P_\rho(V(R))=P(V(R))p(\rho)\ . \tag{3}$$

Such a probability distribution has also been predicted by Goldburg and co-workers [1–3] from their measurements of the intensity correlation function of the scattered light from a seeded flow flowing through a grid at an intermediate turbulence range.

To verify this experimentally, we have done experiments similar to ones described in [1–3] in the medium range of turbulence having Reynolds numbers in the range of 300–800. Here we define the Reynolds number in the usual way as au/ν, where a is the mesh size of the grid, ν is the kinematic viscosity, and u is the mean flow velocity at the center line of the pipe. The outline of the experimental setup is shown in Fig. 1.

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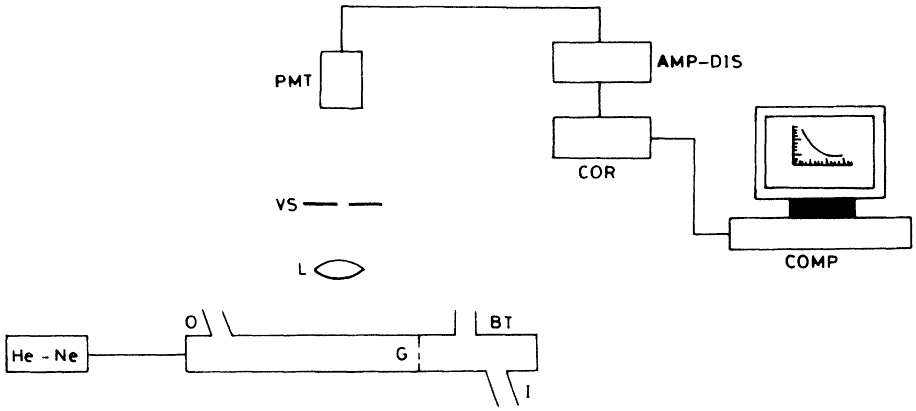

FIG. 1. Outline of the experimental setup. G: grid; I: inlet; O: outlet; BT: bubble trap; L: lens; VS: variable slit; PMT: photomultiplier tube; COR: digital correlator; AMP-DIS: amplifier-discriminator; COMP: computer.

Distilled water is circulated through a glass cell of diameter 3.5 cm by means of a closed pump system. A grid of mesh size 3 mm made of steel wire rods of diameter 1.7 mm placed inside the cell near the high pressure end generates the desired turbulence. Laser light from a 10 mW Spectra Physics He-Ne laser is scattered from the turbulent fluid and focused onto a variable slit with unit magnification before reaching a photomultiplier tube (RCA model 31034A). The output of the PMT after suitable digitization is fed to a 64-channel Malvern Digital photon correlator (7032CN) which is preprogrammed to carry out the autocorrelation analyses. The ACF is sensitive to the size of the eddies controlled by the variable slit width. As the velocity difference $V(R)$ is a random variable, the ACF of the scattered light at the PMT will be the average of the phase factor $\exp[ik\cdot V(R)\tau]$, where $k=(4\pi n/\lambda)\sin(\theta/2)$. If $P(\mathbf{V}(R))$ is the normalized velocity distribution, then the ACF is [5,7]

$$g(\tau)=NA^2\int e^{ik\cdot V(R)\tau}P_p(\mathbf{V}(R))d^3V \; . \tag{4}$$

For the case of isotropic geometry, since $P(V(R))$ is independent of direction, we have

$$P_p(\mathbf{V}(R))=4\pi V^2 P_p(V(R)) \; . \tag{5}$$

Substituting Eq. (5) in Eq. (4) and integrating over the angles, we get [7]

$$g(\tau)=\int \frac{\sin[kV(R)\tau]}{V(R)k\tau}P_p(V(R))\,dV(R) \; . \tag{6}$$

The Fourier sine transform of the above equation yields

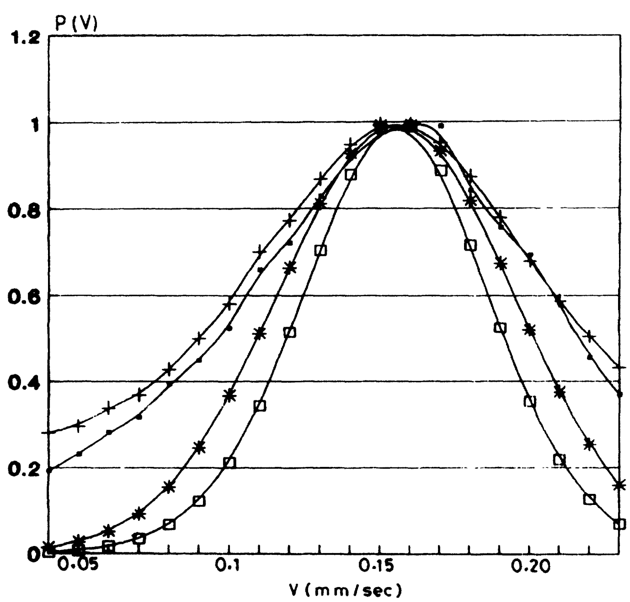

FIG. 2. Probability distribution function for the relative velocity fluctuations extracted from Eq. (7) at Reynolds number ≈300 using experimentally measured values of autocorrelation function (ACF). The points on the curve ■ indicate experimental points, while + indicates Lorentzian, * indicates Gaussian, and □ indicates the product of Lorentzian and Gaussian points.

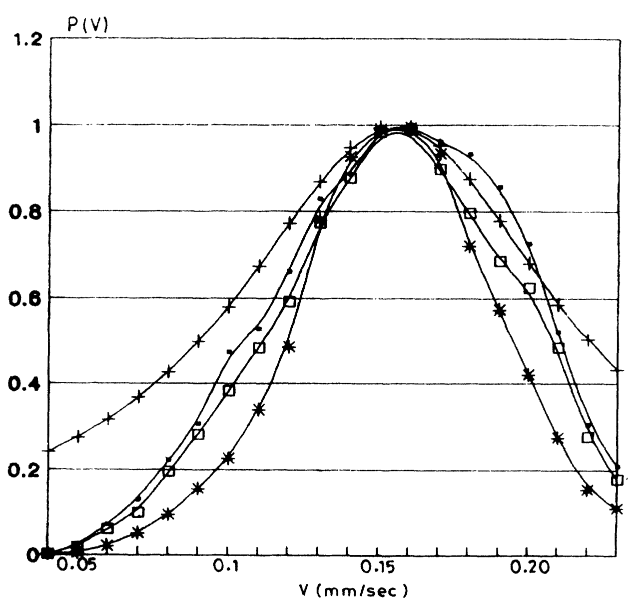

FIG. 3. Probability distribution function for the relative velocity fluctuations extracted from Eq. (7) at Reynolds number ≈500 using experimentally measured values of autocorrelation function (ACF). The points on the curve ■ indicate experimental points, while + indicates Lorentzian, * indicates Gaussian, and □ indicates the product of Lorentzian and Gaussian points.

$$P_s(V)=\frac{2V(R)}{\pi}\int_0^\infty k\tau g(k\tau)\sin[kV(R)\tau]\,d(k\tau)\ . \quad (7)$$

The final distribution of the velocity difference taking into account the slit width becomes

$$g(\tau)=\int_0^L h(R)\,dR \times\int_{-\infty}^{\infty}\frac{\sin[kV(R)]\tau]}{V(R)k\tau}P_\rho(V(R))\,dV(R)\ , \quad (8)$$

where $h(R)$ is the number fraction of particle pairs separated by distance R in the scattering volume [1] and L is the slit width (~ 0.5 mm for our experiments).

Our results for $P_\rho(V(R))$, which have been extracted on a computer using the measured values of ACF in Eq. (7), are displayed in Figs. 2 and 3. The two curves in Figs. 2 and 3 refer to Reynolds numbers of 300 and 500, respectively. We see that for a low Reynolds number, the Lorentzian fit is closest to the experimentally observed curve, and for the medium Reynolds number of 500 it is the product of a Lorentzian and a Gaussian that corresponds to the observed probability distribution function of the relative velocities. The value obtained from our experiments for the ratio of widths of the Gaussian (σ) to Lorentzian (a) are in the range of 2.4–2.8 for the intermediate turbulence range, agreeing with the approximate value of 3 obtained by Pak, Goldburg and Sirivat [2]. Typical values of σ and a obtained from our experiments at the Reynolds number of 500 are $\sigma=1.67\times10^{-4}$ m/sec and $a=0.64\times10^{-4}$ m/sec, giving the ratio $\sigma/a=2.6$. For still higher values of Reynolds number, ≈ 1000, it is the Gaussian which follows the experimental curve. This can be explained perhaps due to the fact that for high values of Reynolds number, the correlation distribution itself tends to a Gaussian rather than a Lorentzian, making the overall $P_\rho(V(R))$ the product of two Gaussians in Eq. (3) and therefore itself a Gaussian.

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